Harmonic mapping

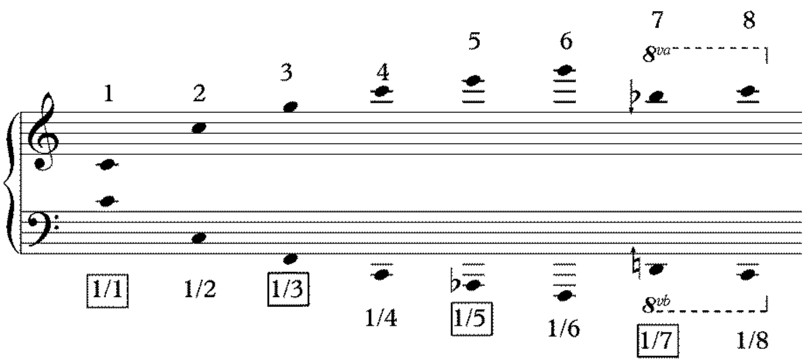
- Uses exclusively rational numbers derived from the harmonic series in the mapping process.
- Extended upon the ideas of **Hugo Riemann**, who had been the first to coin **functional harmony**, its degrees and what their inversions correspond to.
- Prevailing **symmetry**.

The Limit System

- "Music seems to have **advanced in up the harmonic series** throughout history."
- Alois Halba, Harry Partch, Ivor Darreg all suggested that music has been striving to incorporate **higher and higher harmonics for complexity**.
- Discovered Hermann von Helmholtz's *Tonempfindungen* (sensations of tone). It provided him with the scientific explanation for many of the things he was curious about.
- "The scale of musical intervals begins with absolute consonance (1:1) and gradually progresses to an infinitude of dissonance, **the consonance of the intervals decreasing as the odd numbers of their ratios increase."**
- Some sort of homage to Schoenberg's **emancipation of dissonance**.
- Each new prime and odd number composite number generates a new pitch within the overtone series, called the **identity:**

[1] 2 [3] 4 [5] 6 [7] 8 [9] 10 [11] 12 [13] 14 [15]

- 3 id., 5 id., 7id. etc.
- Two other terms to be associated with are **Otonality** (over-tonality) and **Utonality** (under-tonality).
- Partch conceptualized an **undertone series** accompanying the overtone series.

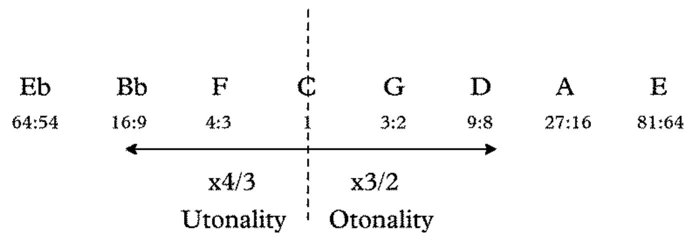


- Every identity in the overtone series (e.g., 3:1, 7:1, 19:1), called the **Odentity** (over-identity), is defined by the **odd number** in its numerator. Conversely, every corresponding **Udentity** (under-identity) in the undertone series (e.g., 1:3, 1:7, 1:19) is defined by the **odd number** in its denominator.
- Partch put forth **Otonality** as some sort of a **major tonality**, the earliest appearance of this sonority in the overtone series is: 1:1, 5:4, 3:2.

- The inversion of this sonority is 1:1, 8:5, 4:3; this yields a minor chord which is also its first appearance in the undertone series, and thus **Utonality** is associated with **minor tonality**.
- The concept of **dual identity**
- The term **limit** was introduced by the American composer Harry Partch.
- Gives an **upper bound** insofar as the complexity of the harmony.

<u>**3-limit** harmonic mapping </u>(Pythagorean)

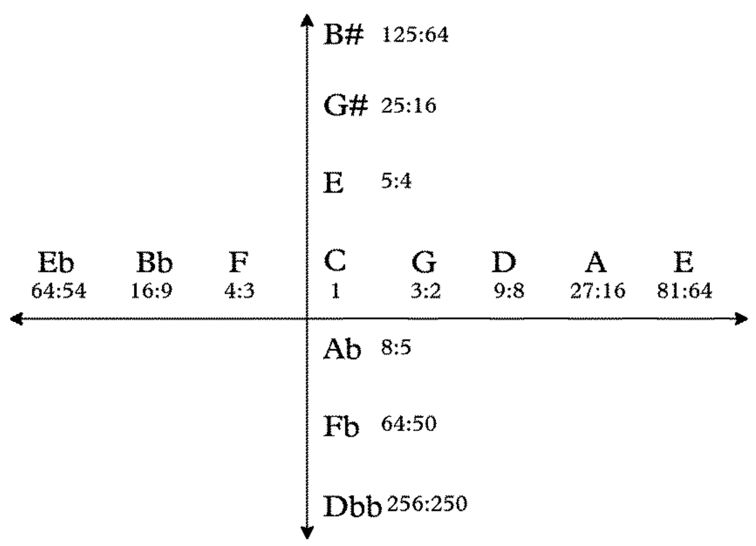
- Takes into account the 3rd partial (fifth) and thus establishes **3:2 ratio** (odentity) and its inversion **4:3 ratio** (udentity) as its basic multipliers (pitch generators).



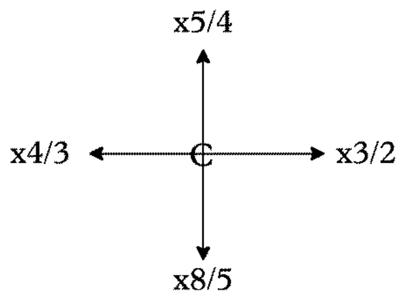
- Notice the domineering symmetry; this phenomenon is due to the notion that intervals here are the inversions of one another.

<u>**5-limit** harmonic mapping</u>

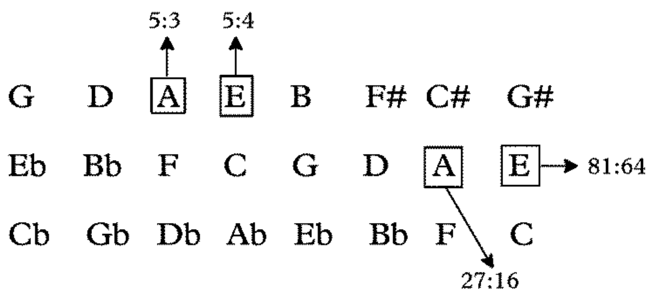
- Takes into account the 3rd and the 5th partials (Odentities) and thus establishes **3:2 and 5:4 ratios** and their inversions (Udentities) **4:3 and 8:5 ratios** as its fundamental multipliers.
- Notice again how each ratio is kept within the bounds 1<x<2



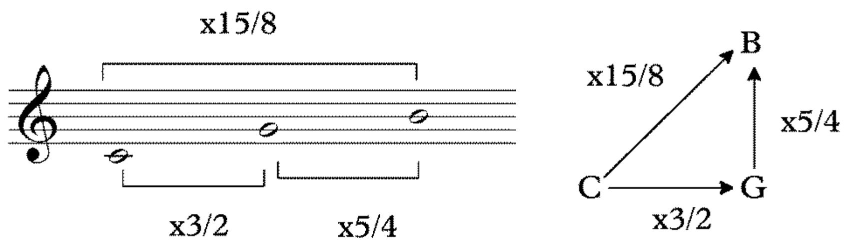
- A comprehensive formula for pitch generation in four directions is then:



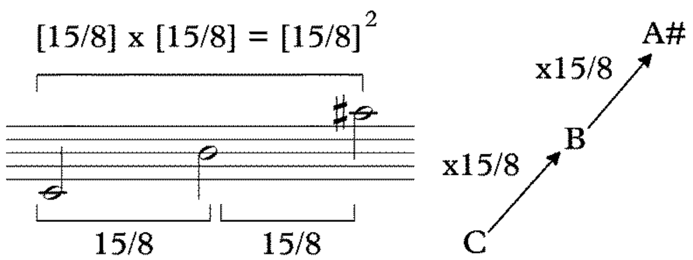
- When each pitch is allowed its own set of multipliers the resulting sequence of notes offers more possibilities in terms of eschewing **undesirable ratios (large numbers):**



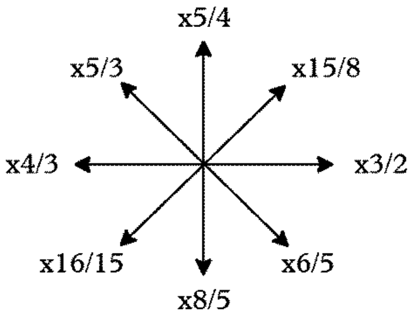
- If C is taken as the reference pitch, a short hand for generating a pitch by diagonal movement would be the stacking up of intervals 3:2 and 5:4, [3/2] x [5/4]:



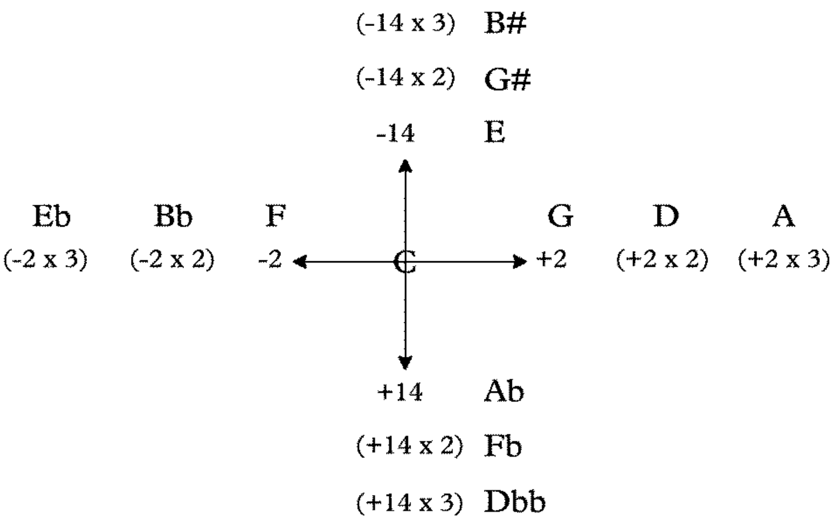
- And so going two steps diagonally would mean staking up the same interval twice [15/8] x [15/8] = $[15/8]^2$



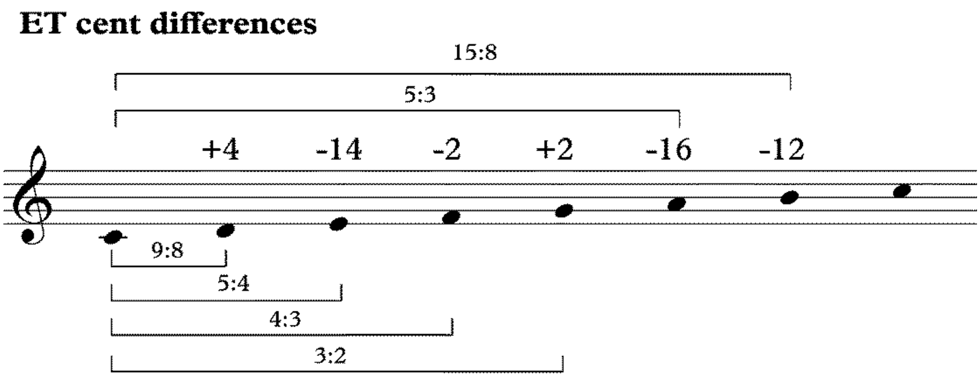
- And so finally a comprehensive formula that takes into account all eight directions is then:



- Let us compare the 5-limit system with equal temperament. The cent differences proceed as follows: basic cent difference multiplied by n, where n is the number of steps taken from the initial pitch (at zero cents).



- One scale derived from the 5-limit system has the following cent differences when compared to a similar scale from ET:



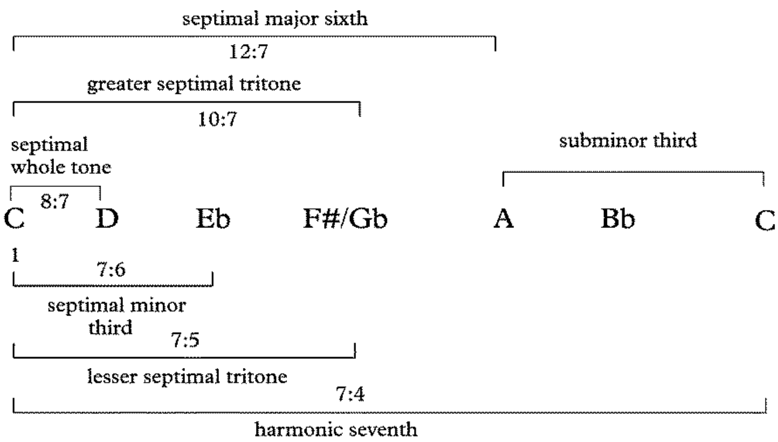
Septimal tuning (not 7-limit yet!)

- Takes into account only the natural 7th partial (identity), however unlike the systems described above it does not use the expected 7/4 ratio as a multiplier (pitch generator). Because already the first step would yield an undesirable ratio, namely 7:4 x 7:4 = 49:32.
- Instead it begins with a harmonic seventh (7:4) and divides it into a tritone (7:5) and a minor third (7:6).
- Then the inversions of those are incorporated in the opposite direction (8:7), (10:7) and (respectively).

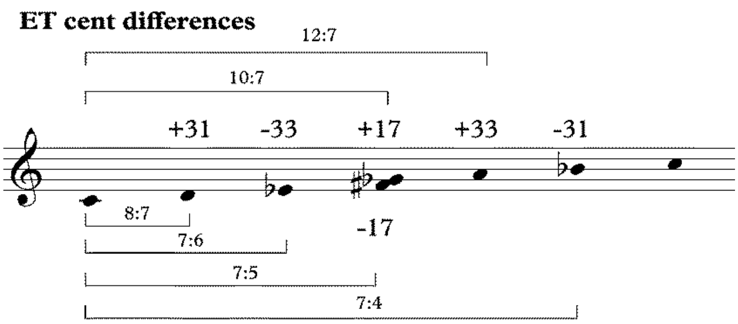
D F# A [C] Eb Gb Bb

Udentities Odentities

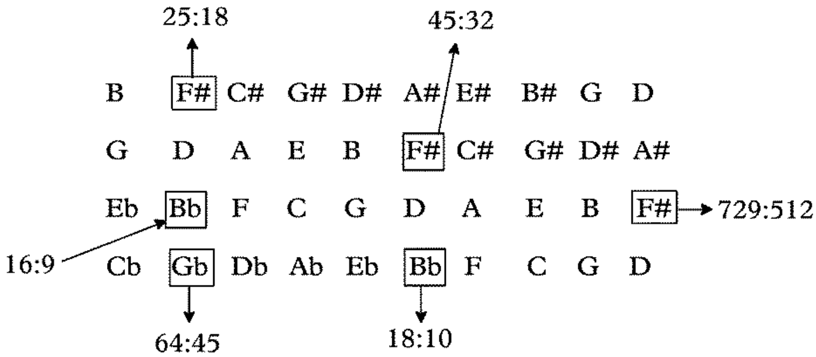
- Put in scalar form:



- Compared to ET, we have the following cent differences:



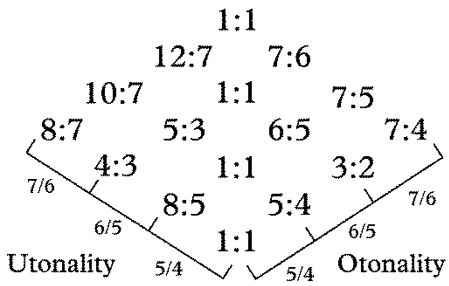
- This tuning system may not be overly desirable by itself in terms of achieving **just intonation,** however when merged 5-limit, what we have is another dimension and therefore more options in terms of just intervals.
- In 5-limit tuning, we have several less-than-stellar tritones and minor sevenths:



- That is why 7:5 and 10:7 are good substitutes for tritones and 7:4 for minor seventh, because they are simpler in terms of mathematical expression, and therefore more desirable (more just).

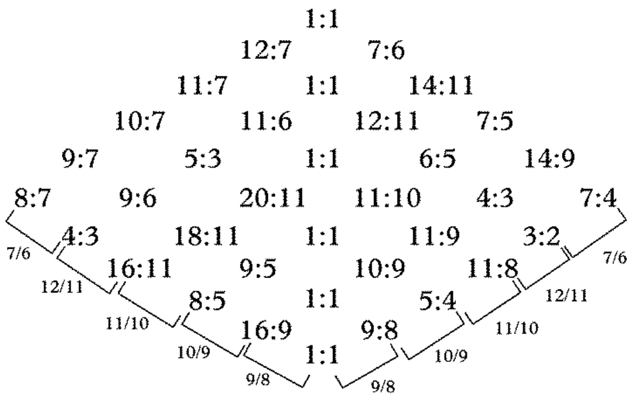
<u>**7-limit** harmonic mapping</u>

- This system, as mentioned above, simply adds septimal tuning as a dimension to the 5 limit system.
- Below is Partch's 7-limit tonality diamond:



<u>**11-limit** harmonic mapping</u>

- This tuning system shares the same principles, but of course because of the addition of higher identities (1, 3, 5, 7, 11), it incorporates more options in terms of available ratios to be used.



- An 11 note scale can be devised by combining the first otonality with the first utonality:

0	203.9	231.2	386.3	498.0	551.8	648.7	702.0	813.7	986.8	996.1
1:1	9:8	8:7	5:4	4:3	11:8	16:11	3:2	8:5	7:4	16:9

- Partch then derives a 29-note scale by using the ratios found in 11-limit diamond above:

cents	0	150.6	165.0	182.4	203.9	231.2	270.0	315.6	347.4	386.3	417.5	435.1	498.0	551.3	582.7	617.5
	1:1	12:11	11:10	10:9	9:8	8:7	7:6	6:5	11:9	5:4	14:11	9:7	4:3	11:8	7:5	10:7

648.7	702.0	765.0	782.5	813.7	852.6	884.4	933.1	968.8	996.1	1017.6	1035.0	1050.0	
16:11	3:2	14:9	11:7	8:5	18:11	5:3	12:7	7:4	16:9	9:5	20:11	11:6	2:1

- This scale for the most part proceeds in **20-30 cents**, and as it can be observed, the cent difference between the first two notes is already larger than a semitone (100 cents).
- Moreover there are gaps elsewhere; a better, more consistent scale can be constructed if some of the gaps are filled: between 1:1 and 12:11, 7:6 and 6:5, 9:7 and 11:8, 16:11 and 14:9, 5:3 and 12:7, 11:6 and 2:1

1:1 0	syntonic comma 81:80 21.5	33:32 53.2	septimal chromatic semitone 21:20 84.5	just semitone 16:15 111.7	12:11 150.6
7:6 267	32:27 294	6:5 315.6			
9:7 435.1	narrow fourth 21:16 470.8	4:3 498.0	5-limit impure fourth 27:20 519.5	11:8 551.3	
16:11 648.7	5-limit impure fifth 40:27 680.5	3:2 702.0	wide-fifth 32:21 729.2	14:9 764.9	
5:3 884.4	pythagorean sixth 27:16 905.9	12:7 933.1			
11:6 1049.4	just major seventh 15:8 1088.3	diminished octave 40:21 1115.5	64:33 1146.8	syntonic-comma diminished octave 160:81 1178.1	2:1

- In the above figure the **21:20 ratio** is a **septimal chromatic semitone**, and could be thought as the difference between the 21st (7 x 3) and the 20th partials.
- **16:15 is a just semitone** and it corresponds to the difference between the perfect fourth and the major third: [4:3] / [5:4] = 16/15 and that of a perfect fifth and a minor

sixth: [8:5] / [3:2] = 16:15. Moreover and perhaps more importantly, this ratio is the inversion of a just Major seventh (15/8).

- **33:27 is a Pythagorean minor third** that is encountered between D and F in the C major diatonic scale: [4:3] / [9:8] = [4:3] x [8:9] = 32:27.
- **40:27 is an impure fifth** that can arise in a 5-limit tuning system between D and A in a C major diatonic scale: [5:3] / [9:8] = [5:3] x [8:9] = 40:27.
- **And so 27:20 is the impure fourth** which is simply the inversion of 40:27.
- The resultant system is called **43-note per octave scale** designed by Harry Partch.
- Listen to Partch's voice.

[Accompanying multimedia: Youtube documantaries]