Just intonation

- Generating ratios using rational numbers from the harmonic series.
- Rational numbers are ratios of **integers**.
- Acoustically simpler sounding.

Comparison with equal temperament

Unison $2^{0/12} = 1.000000$ 0 **1** = 1.000000

Minor second $2^{1/12} = \sqrt[12]{2} = 1.059463$ +11.73 **16/15** = 1.06666

Major second $2^{2/12} = \sqrt[6]{2} = 1.122462$ +3.91 **9/8** = 1.1250000

Minor third $2^{3/12} = \sqrt[4]{2} = 1.189207$ +15.64 **6/5** = 1.2000000

Major third $2^{4/12} = \sqrt[3]{2} = 1.259921$ -13.69 **5/4** = 1.2500000

Perfect fourth $2^{5/12} = \sqrt[12]{32} = 1.334840$ -1.96 **4/3** = 1.33333

Tritone $2^{6/12} = \sqrt{2} = 1.414214$ -17.49 **7/5** = 1.4000000

Perfect fifth $2^{7/12} = \sqrt[12]{128} = 1.498307$ +1.99 **3/2** = 1.5000000

Minor sixth $2^{8/12} = \sqrt[3]{4} = 1.587401$ +13.69 **8/5** = 1.6000000

Major sixth $2^{9/12} = \sqrt[8]{8} = 1.681793$ -15.64 **5/3** = 1.66666

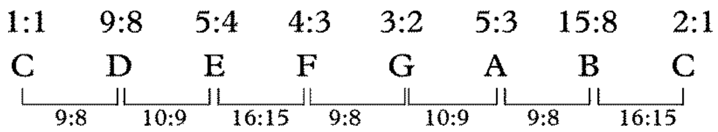
Minor seventh $2^{10/12} = \sqrt[6]{32} = 1.781797$ -31.18 **7/4** = 1.77777

Major seventh $2^{11/12} = \sqrt[12]{2048} = 1.887749$ -11.73 **15/8** = 1.8750000

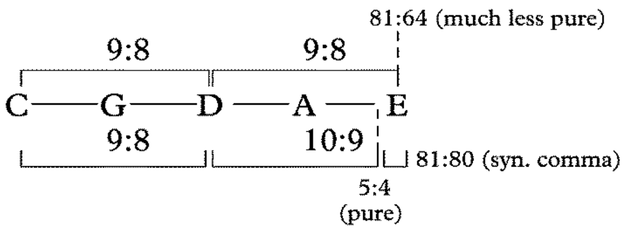
Octave $2^{12/12} = 2 = 2.000000$ 0 **2/1** = 2.000000

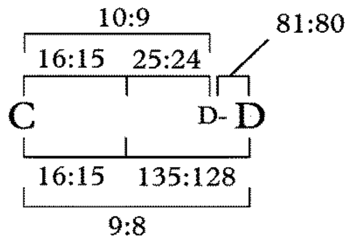
The necessity of the syntonic comma in Ben Johnston’s music

- All of Johnston’s work in JI is built with C as the reference point (1:1).
- The basic entity that arises from this reference point is the just C Major scale:

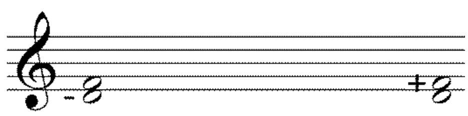


- In this scale C-E-G and G,B,D are just major triads and already available for usage
- In just intonation, the syntonic comma (a distance of about 22 cents) is necessary to keep some of the intervals justly tuned.

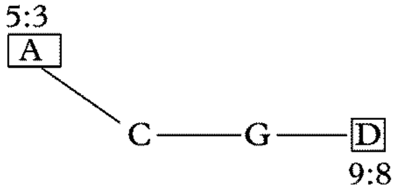




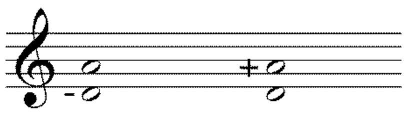
- The **syntonic comma** could be thought as the **difference between the two components of the 5:4 pure major third**; 9:8 and 10:9 ([9:8] / [10:9] = 81:80).
- The 9:8 and 10:9 have in turn their own components: the just semitone **16:15** and 135:128, which completes it to 9:8 and also 25:24, which completes it to 10:9
- The difference between these two complementary intervals is again the syntonic comma ([135:128] / [25:24] = 81:80).
- In this sense discrepancies pertaining to the syntonic comma arise because some of the pure intervals like 5:4 and 6:5 are constructed with different sized whole tones and semitones, whose differences in sizes amount to 81:80.
- For example: the difference between D and F in 5-limit tuning does not amount to a 6:5, but [4:3] / [9:8] = 32:27, this is **smaller** than a 6:5 also by a syntonic comma! [32:27] x [81:80] = 6:5
- Therefore, if we use an F from the just intonation C major scale, and tune a D to it, then the D must be lowered by the syntonic comma in order to obtain a just minor third. Similarly, if we use the D from the same scale and tune an F to it, then the F must be raised by the syntonic comma in order to obtain a just minor third:



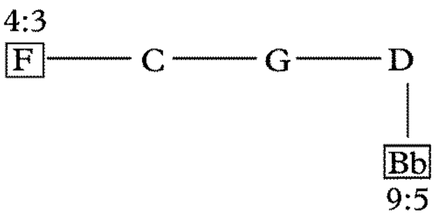
- Here is another example:

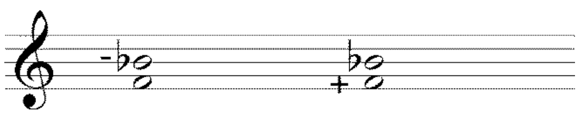


- In this case, the difference between 5:3 and 9:8 is [5:3] / [9:8] = 40:27, which is **smaller** than a 3:2 perfect fifth by the syntonic comma: [40:27] x [81:80] = 3:2.
- Therefore, if we use an A from the just intonation C major scale, and tune a D to it, then the D must be lowered by the syntonic comma in order to obtain a just perfect fifth. Similarly, if we use the D from the same scale and tune an A to it, then the A must be raised by the syntonic comma (27:16) in order to obtain a just perfect fifth:

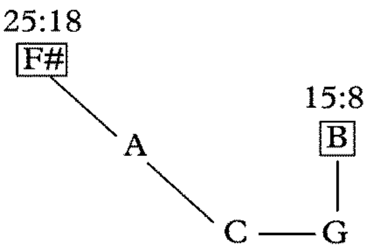


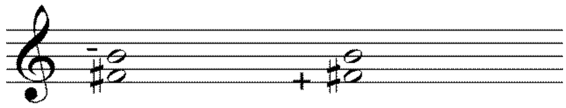
- Here, the difference between the two intervals do not amount to the expected 4:3: [9:5] / [4:3] = 27:20, which is **larger** than 4:3 by the syntonic comma: [27:20] / [81:80] = 4:3

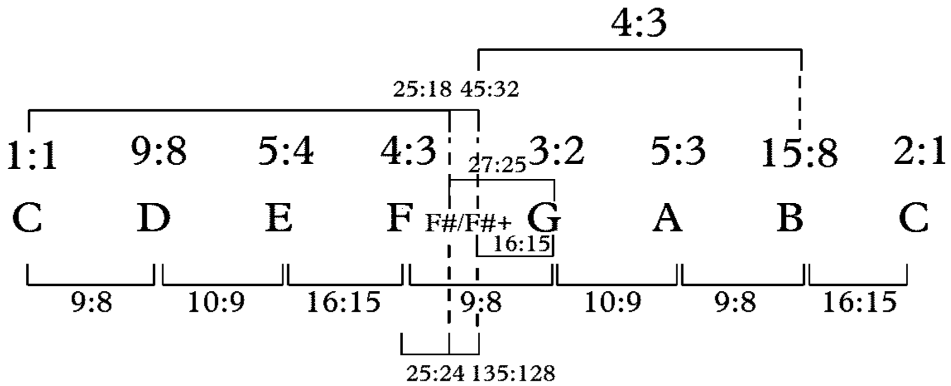




- Therefore, if we use an F from the just intonation C major scale, and tune a Bb to it, then the Bb must be lowered by the syntonic comma in order to obtain a just perfect fourth. Similarly, if we use a just 9:5 Bb and tune an F to it, then the F must be raised by the syntonic comma in order to obtain a just perfect fourth.
- The same must also be done for the discrepancy that arises between D and F# and F# and B:







- The F# used in the above diagram is 25:18, had this been used in conjunction with D – (10:9), it would create a just 5:4 ([10:9] x [5:4] = 25:18). But if it is used in conjunction with the D (9:8) from the JI C major scale, then it must be enlarged by the syntonic comma ([9:8] x [5:4] = [25:18] x [81:80] = 45:32)
- Similarly, when it is used in conjunction with B from the JI C major scale, it must be enlarged by the syntonic comma to obtain 45:32.
- It is important to note now how the F# is actually obtained: **25:24 (70 cents)** is the difference between JI 5-limit minor-major thirds ([5:4] / [6:5] = 25:24) and also JI 5-limit minor-major sixths ([5:3] / [8:5] = 25:24). **Added to any diatonic note, it produces the # version of that note.** The 25:18 F# is generated this way ([4:3] x [25:24] = 25:18)
- Let us now derive some more major chords using the 25:24 interval:

D F#+ A

A C# E

E G# B

B D# F#+

C# is [1:1] x [25:24] = 25:24

G# is [3:2] x [25:24] = 25:16

D# is [9:8] x [25:24] = 75:64

F# is [4:3] x [25:24] = 25:18

Derivation of the flats

- Using the utonal versions of some commonplace ratios we obtain:

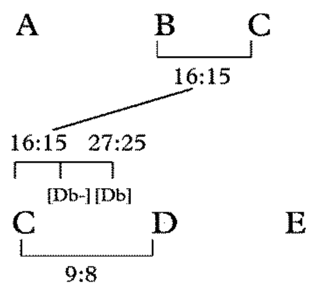
Eb is 6:5 (inverted A, which is 5:3)

Ab is 8:5 (inverted E, which is 5:4)

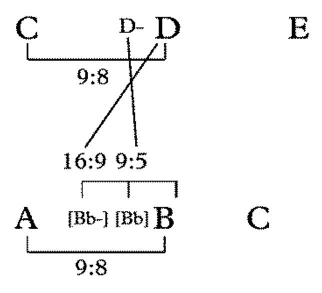
Db- is 16:15 (inverted B, which is 15:8), therefore Db is [16:15] x [81:80] = 27:25

Bb- is 16:9 (inverted D, which is 9:8), therefore Bb is [16:9] x [81:80] = 9:5

- The reason 16:15 is a Db- is because when B is inverted (8:15), we need a distance of a **M7 down from C**, the Db encountered in that octave needs to be as much flattened as the B up from C is sharpened.



- The same phenomenon applies to Bb-.

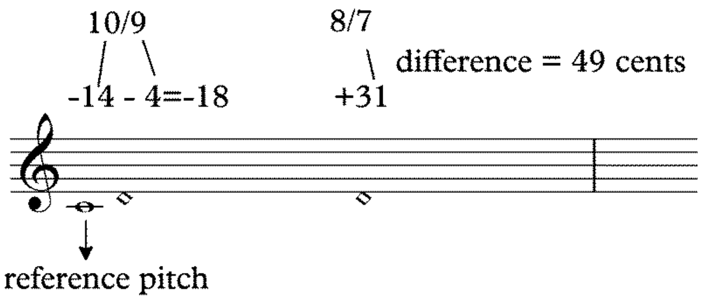


- This covers all of the intervals in a JI 5-limit scale:

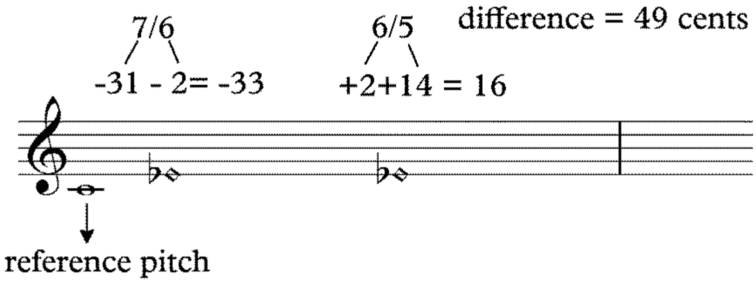
note	ratio	cents
C	1/1:	0
C#	25/24	70
C#+	135/128	92
D♭–	16/15	112
D–	10/9	180
D	9/8:	204
D#	75/64	275
E♭	6/5	316
E	5/4	386
E+	81/64	408
F♭	32/25	428
F	4/3:	498
F+	27/20	520
F#+	45/32	590
G♭	36/25	632
G	3/2:	702
G#	25/16	772
A♭	8/5	814
A	5/3:	884
A+	27/16	906
A#+	225/128	976
B♭–	16/9	996
B♭	9/5	1018
B	15/8:	1088
C♭	48/25	1130
C	1/1	0

<u>**Other Intervallic adjustments in Ben Johnston's music**</u>

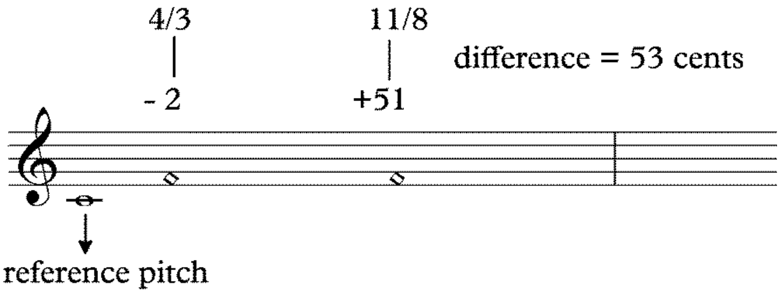
- These adjustments are calculated in terms of **deviations from ET** and are necessary to fix **discrepancies that arise between JI and ET**.
- **36:35 (49 cents):** known as the septimal quartertone (7th partial relation).
- This interval amounts to the difference between **two whole tones: 10:9 and 8:7:**



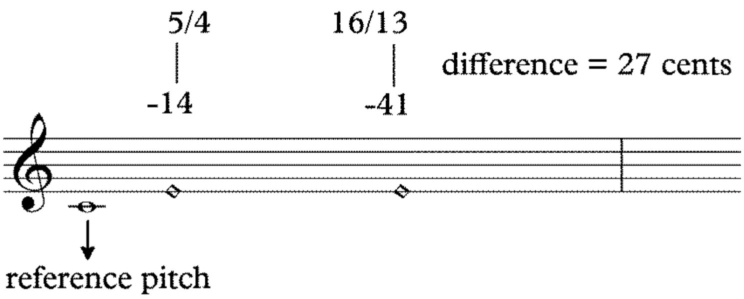
- It also amounts to the difference between **two just minor thirds 7:6 and 6:5:**



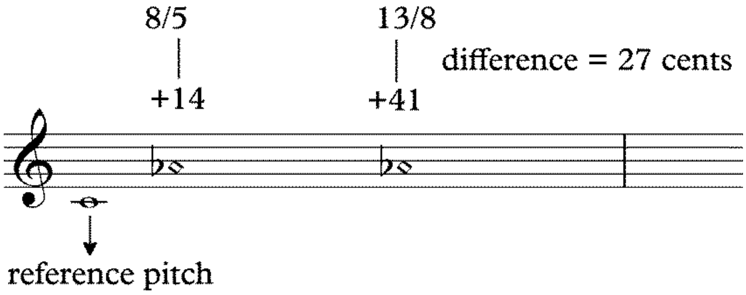
- And finally, this same interval amounts to the difference between 5:4 and 9:7, 14:7 and 8:5, 5:3 and 12:7, 7:4 and 9:5.
- **33:32 (53 cents):** known as the **al-Farabi quartertone** (11th partial relation).
- This interval is the exact amount needed to raise a just perfect fourth to get a just tritone ([11:8] / [4:3] = 33:32).



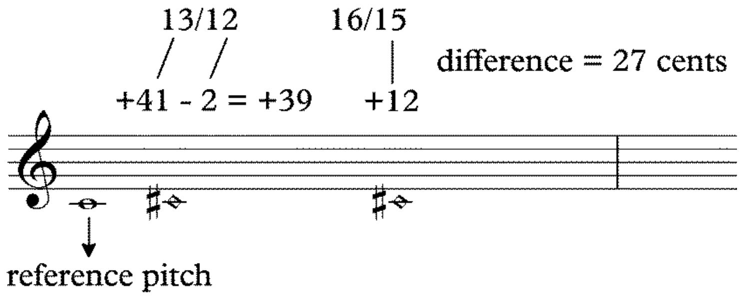
- **65:64 (27 cents)**: known as the **13th partial chroma** (13th partial relation).
- It amounts to the difference between a **tridecimal neutral third (359 cents) and a just third (386 cents).**



- It also amounts to the difference between a **tridecimal neutral sixth (841 cents) and a just minor sixth (814 cents).**



- This interval amounts to many other intervallic differences, here is one more: **a 5-limit just semitone (112 cents) and a tridecimal subtone (139 cents).**



- Much more open than Partch's systems, allowing for pitch center changes and modulations.
- Extended the limit system to beyond 11.