## Basic Idea of a temperament

- Temperament occurs to date from the 1400s and earliest practical discussions come from the following century.
- In a tuning system that accommodates more than one type of interval, a dominating interval is taken as a basis and used to derive the rest.
- Arose because of keyboard instruments.
- What is temperament?
- Wrenching/tempering intervals from their pure configurations.
- Process of compromise whereby the acoustical **purity of one interval is jeopardized** (as little as possible) in order to **lessen the acoustical impurity of another interval**.
- Specifically speaking it is the process of changing the size of **the fifth** in order to accommodate **the major third**.
- Making thirds sound good and pure **(5/4, 386.31cents)**, whereas compromising the purity of the fifth.

## Pythagorean Tuning (not temperament!)

- Pythagorean tuning is not a temperament because it uses stacked un-tempered fifths.
- The acoustically pure fifth **3:2 (702 cents),** a fundamental building block of medieval music.
- Pythagoras was significant because he devised a numerical basis for acoustics.
- Attributed to Pythagoras, for his alleged discovery of the 3:2.
- In fifths:

| | | | | | | |
|---|---|---|---|---|---|---|
| F | C | G | D | A | E | B |
| 2:3 | 1 | 3:2 | 9:4 | 27:8 | 81:16 | 243:32 |

- Scalar:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| C | D | E | F | G | A | B | |
| 1 | 9:8 | 81:64 | 4:3 | 2:3 | 27:16 | 243:128 | (keeping the ratios 1<x<2) |

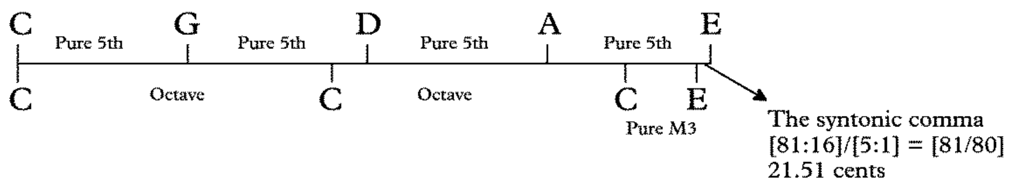
The semitone in this system would be calculated by subtracting two whole tones from the original fourth. (4:3/9:8)/9:8 = 256/243

Problems:

- Octave overshooting at the completion of the circle of fifths

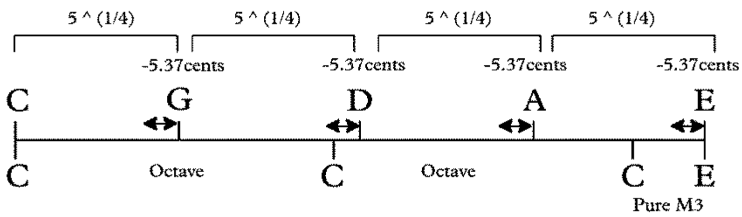
(3:2) ^ 12 = 129.746
(2:1) ^ 7 = 128

- 129.746/128 = **1.014:1** (about quarter of a semitone discrepancy, **24.06 cents**)
- Extremely wide major third compared to a pure third (5:1)




**Quarter-comma meantone**

- Most common tuning system in the Renaissance.
- First encountered in Pietro Aron's *Toscanello* (1523)
- Another name to receive credit is Gioseffo Zarlino.
- A successful attempt at obtaining some **pure major thirds (386.31 cents)**.
- Starting with pure major thirds is not satisfactory, because going up three major thirds does not amount to an octave: $5/4^3$ = 125/64 = 1.9531:1 instead of the 2:1.
- This might not seem like a lot but in fact through tempering the octave by this amount we can find the exact ratio of this discrepancy: [2]/[125/64] = **128/125**, this interval is called **the diesis** and is as wide as a quartertone! **(41.0588 cents).** This interval is much larger than the pure-fifths-octave-overshoot we will see later.
- We must rely on our fifths and derive everything else from them.
- Each of the four perfect fifths needs to be tempered **by one quarter of the overall discrepancy**, which is the syntonic comma **(21.50629 cents)**. Thus 21.50629/4 = **5.3765 cents**.
- Thus 702 – 5.3765 = 696.6235 cents
- This is where the tuning system gets its name from.

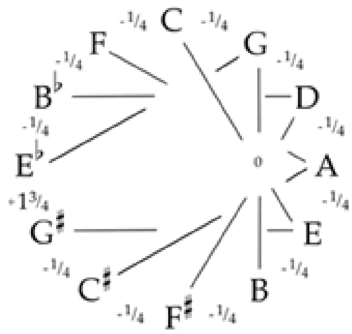



- When this cent value is converted into a frequency ratio: 696.6235 = 1200. $\log_2(x)$, $x$ = 1.49539 = **the fourth root of five** ($\sqrt[4]{5}$)
- Since the stacking of four tempered fifths has to equal to a 5:1 ratio (5$^{th}$ partial), what is the ratio of each fifth? $x^4$ = 5, x = $\sqrt[4]{5}$
- Another way to go about this is by achieving the temperament through the syntonic comma ratio, which is **81/80**. In order to divide this interval into four equal parts we take its fourth root: $\sqrt[4]{81/80}$ and so this is the exact amount a 3/2 ratio needs to be tempered by: [3:2]/ [$\sqrt[4]{81/80}$].

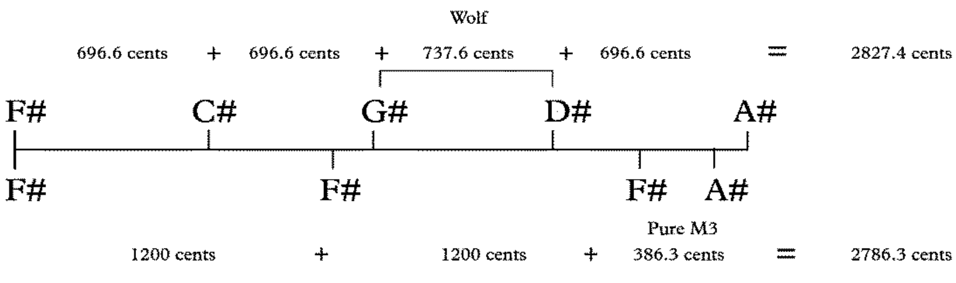
- **The whole tone in this system (D) is exactly at the halfway point from the beginning C and the E (two tempered fifths instead of four). This gives a ratio of $[\sqrt{5}]/2$.**
- Continued until the end of the seventeenth century.
- Still persisted until the time of Wagner.

Problems:

- Wolf: The ET perfect fifth spans exactly **700 cents** and the tempered fifth in this tuning system spans about **696.6235 cents**, yielding a difference of **about 3.3765 cents.** Since there are eleven tempered fifths encompassing seven octaves, there is going to be **37.1415 (3.3765x11) cents** of discrepancy between the penultimate and the last fifth completing the octave. So the wolf is 700 + 37.1415 = **737.1415 cents**, significantly sharper than the pure fifth (approximately 702 cents).
- As can be observed from the figure below, any tempered cycle of four fifths that traverses the wolf point (in this case G# to Eb), will result in not in perfect thirds or even anything that could be called a third, but diminished fourths.



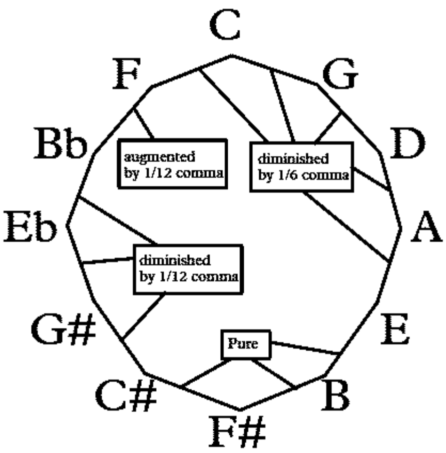

- It consists of eight acoustically perfect thirds and however there are also **four really disturbing ones (diminished 4ths).**
- Let us calculate one for F#-A#:




- As can be observed from the figure above there exists 2827.4 – 2786.3 = 41.1 cents of discrepancy between the acoustically pure major third and the other one that traverses the wolf on its path. This results in an interval that cannot even be named a third, but a diminished 4th **(427.4 cents)**.

**Intervening years**

- In the sixteenth century theorists began to experiment with variants of quarter-comma meantone
- Usually by diminishing the amount of tempering by spreading the syntonic comma discrepancy over more fifths (1/5, 1/6).
- Some of the important names to be credited are Micheal Praetorious and Pier Francesco Tosi.
- Making fifths not as narrow by spreading the comma over five or six of them.
- For example in sixth-comma meantone the amount that fifths are tempered by is $\sqrt[6]{81/80}$. This causes the new fifth to be **698.37341 cents**, causing **1.62659 cents** of discrepancy for every narrowed fifth (Taking ET as a reference point)**.** This value spread over eleven fifths is 11 x 1.62659 = **17.89 cents**. This is the amount of discrepancy between the penultimate fifth and the last one that completes the octave, and so the wolf fifth amounts to about **717.89 cents**, significantly better than the wolf fifth in the quarter-comma meantone (737.1415 cents).
- New irregular systems began to be popular in the late seventeenth century.



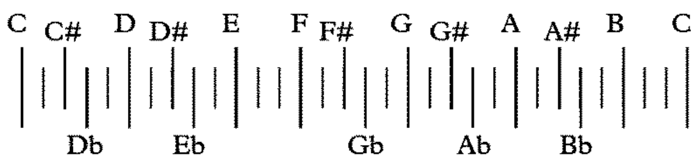

- These systems juxtaposed different amounts of temperaments of fifths and thirds throughout.
- Alexander Werckmeister, Johann Philipp Kirnberger, Johann Georg Neidhardt.

**Equal Division**

- Meantone temperaments result in two kinds of semitones. Greater and lesser.
- In a quarter-comma meantone temperament fifths are the size of $\sqrt[4]{5}$. A diatonic semitone in this system, which is derived by going up three octaves and down five perfect fifths has a ratio of $[2^3]/[5^{5/4}]$.
- In the same system, a whole tone, which is derived by going up two fifths and down an octave has a ratio of $\sqrt{5}/2$ **(117.1 cents).**

- Unfortunately two semitones multiplied ($s^2$) do <u>not</u> equal a whole tone (t) in the same system. Therefore the value of $s$ needs to be multiplied by another number, x, (a second semitone) in order for us to obtain the whole tone: x = t/s = $5^{7/4}/16$ **(76 cents)**.
- Theorists devised methods that accommodated this discrepancy consistently into systems that divided the octave into equal parts.
- **They all feature a common ground in that they divide the whole tone into an odd number of parts. This way a more precise way of dividing the semitones became possible.**
- First introduced by Nicola Vicentino, a modifier of keyboard instruments and an important associate of Carlo Gesualdo. He devised a 31-part scale:


- Five parts are assigned to each whole tone and three to each semitone. There are five whole tones and two semitones, so (5x5) + (2x3) = 31.
- It is important to note that the semitones are the larger ones described above (117.1 cents), for this reason they are also called **diatonic** semitones.
- **The chromatic semitones are indispensible in the attainment of good major thirds** because sometimes, as in the case of D-F#, an amount of 76 cents is needed, rather than 117.1 cents, in order not to overshoot a pure third:
- D-E has a fixed whole tone value of **193.1 cents** and E-F has a diatonic semitone value of **76 cents.** These two numbers add up to: 193.1 + 76 = 310.2 cents. If we add a diatonic semitone to this we get: 310.2 + 117.1 = 427.3 cents, much wider than a pure third. Therefore, we need to add a chromatic semitone as our last step which gives: 310.2 + 76 ≈ 386.31 cents, a pure third.




- This and other equal division systems were each designed to accommodate a particular meantone system:

| Octave divisions | Whole tone divisions | Major semitone divisions | Minor semitone divisions | Meantone equivalent |
|---|---|---|---|---|
| 19 | 3 | 2 | 1 | Third-comma meantone |
| 31 | 5 | 3 | 2 | Quarter-comma meantone |
| 43 | 7 | 4 | 3 | Fifth-comma meantone |
| 55 | 9 | 5 | 4 | Sixth-comma meantone |
| 67 | 11 | 6 | 5 | Seventh-comma meantone |
| 79 | 13 | 7 | 6 | Eight-comma meantone |

- Remember from the Fundamentals part that the *n*th root of a ratio divides a particular interval into *n* number equal parts.
- And remember also that raising a ratio to the *n*th power stacks up an interval *n* number of times.
- Using these two principles we can obtain the ratio value of any interval in any of the above-mentioned systems.
- For example, let us calculate the ratio value of a sixth-comma meantone third.
- First we would need to deduce what equal-division system accommodates the sixth-comma meantone temperament, and the answer is 55-part.
- Let us now divide the octave into 55 equal parts: $\sqrt[55]{2}$. Since a whole tone in this particular system is made up of nine parts, a major third will be made up of 2 x 9 = 18 parts.
- Let us now stack up $\sqrt[55]{2}$ eighteen times in a row: $\sqrt[55]{2}^{18}$ or $2^{18/55}$.
- The formula for any of the intervals in any of the above mentioned systems and temperaments is:

$$2^{\left(\frac{the\ number\ of\ parts\ an\ interval\ encompasses}{total\ number\ of\ parts}\right)}$$

**<u>Equal temperament</u>**

- ET strives to establish all equidistant intervals that in turn add up to the octave.
- Let us take into account fifths; normally twelve acoustically perfect fifths do not correspond in ratio value to seven octaves:

(3:2) ^ 12 = 129.746
(2:1) ^ 7 = 128

- The resultant discrepancy is divided by twelve and that is how each fifth is tempered: 129.746:128 = 1.014:1 corresponds to about **24 cents**, divided by 12, is 2 cents. Each fifth in the ET is tempered by about 2 cents.
- Each semitone is also equidistant in that the octave is divided into all equal multiples: $\sqrt[12]{2}$.

<u>Problems:</u>

- Very wide thirds 1.25992 as opposed to 1.2500 (5/4)